Normal Distribution

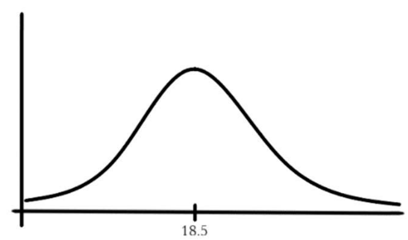
$$P(X=x)=\frac{e^{\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)}}{\sigma\sqrt{2\pi}}$$

This is the probability density function of the Normal distribution. $P(X=x)$ tells us we are dealing with a probability when x is a certain value. x is the number of times the event occurs, μ is the mean (the average), σ is the standard deviation[1], e and π are mathematical constants (numbers with fixed values, 2.71828… and 3.14159… respectively.)

The other two distributions that we have covered are the Poisson and Binomial distributions. The former is for random events with a very small probability of happening (e.g. *"Telephone calls arrive at a call centre on at the rate of 50 per hour. Find the probabilities of 0, 1 or 2 calls arriving in any 6 minute period."* would be a typical Poisson question, the latter is for instances were there can only be success or failure (e.g. *"Seeds have a probability of germinating of 0.9. If six seeds are sown what is the probability of five seeds germinating?"* would be a typical Binomial question (germinating or not germinating are the success or failure).

The Normal distribution is unlike the above two distributions, as it is continuous. This means that items can have any value within a given range[2], e.g. the height of a person can be 1.66m, 2.0956m, or any other value to as many or as few decimal places. The Binomial and Poisson distributions are discrete, so can only take specific values, e.g. number of sweets in a jar, can only be 0,1,2 etc. – you cannot have 1.66 sweets or 2.0956 sweets. One could argue that if you split sweets then you can in fact have 1 and 2/3 of a sweet (1.66 sweets) – but if you allow this then you have made the data continuous.

It is important to note that there are an infinite number of possible values that there can be, and so the probability of any one specific value being the case is 0. So if we want to find the chance of $x=5$, for example, we would need to take a range of values – this is usually 0.5 either side or $4.5<x<5.5$ so that there is an area under the normal distribution graph. To the right is an example graph. The peak is at the mean for the example below.



For example, an industrial process mass produces an item whose weights are normally distributed with mean 18.5kg and standard deviation 1.5kg. We need to find the probability that an item chosen at random weighs 21.5kg.

$$P(X=21.5)=\frac{e^{\left(-\frac{1}{2}\left(\frac{21.5-18.5}{1.5}\right)^2\right)}}{1.5\times\sqrt{2\pi}}$$
$$P(X=21.5)=0.0359\ldots$$

In reality, this computation is usually done on a calculator, on which you can type in the mean, standard deviation and x value and it will churn out an answer for you.

See also

[1] this is a measure of the average difference of a value from the mean. In an exam this will probably be a small single-digit value.
[2] Though in a typical Normal distribution there is no defined range, there will be an implied range – e.g. a person cannot have negative height, even if there may be a tiny probability of this happening according to the model

- Binomial Distribution
- Poisson Distribution

References

Graham, C., Graham, D. and Whitcombe, A. (1986). *A-level Mathematics*. London: Charles Letts. p.154.